Global analysis of Organic Rankine Cycles integrating local CFD simulations and uncertainty

Pietro Marco Congedo*, Jean-Paul Thibault**, Christophe Corre**
Gianluca Iaccarino***

*INRIA Bordeaux Sud-Ouest (BACCHUS Team)
**LEGI-Grenoble
*** Stanford University

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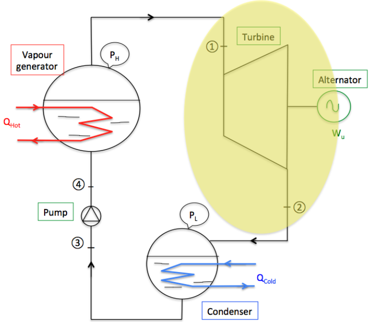
Existing know-how

Local CFD analysis

- Accurate CFD solver for dense-gas flows in a turbine cascade[a] and post-processing of CFD outputs for cycle performance analysis

[a] P.M. Congedo *et al*, Numerical investigation of dense-gas effects in turbomachinery, *Computers & Fluids 49 (2011) 290-301*



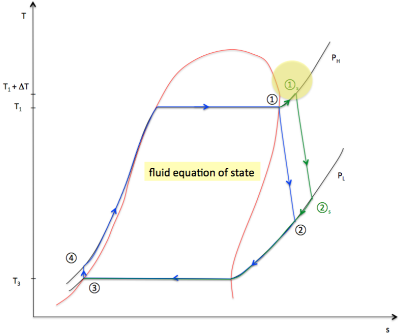
Limitation of the available local CFD analysis

- CFD simulations and turbine design should be performed during cycle design
  → integration of a **local** analysis (during the expansion stage) into the **global** cycle analysis

Other limitations of the available local CFD analysis

- multiple (physical and modeling) sources of uncertainty exist : operating conditions, heat source temperature, thermodynamics . . .
  → must be taken into account for **robust design** and **predictive** simulation
  → need for an efficient stochastic method to propagate uncertainty
  = **non-intrusive polynomial chaos**[a]

[a] P.M. Congedo *et al*, Shape optimization of an airfoil in a BZT flow with multiple-source uncertainties, *Comput. Methods Appl. Mech. Engrg. 200 (2011), 216-232*



Proposed innovative ORC analysis

- Step 1 : coupling UQ tools and local CFD approach
- Step 2 : extending UQ to the whole cycle
  $\rightarrow$ mean value and standard deviation of performance indexes are made available
- Today's presentation focused on Step 1
  $\rightarrow$ robust analysis of a syloxane flow in a turbine cascade
- Detailed perspectives provided for Step 2

- Cell-centered third-order finite volume formulation
- Accommodating an arbitrary EoS (here PRSV)
  with uncertain parameters
- Non-reflecting (characteristic-based) inlet & outlet boundaries
  with fluctuating inlet conditions
- Wall slip condition using multi-D linear extrapolation from interior points to calculate the wall pressure

Mathematical formulation

- Consider the computational problem for an output of interest $u(\mathbf{y}, \boldsymbol{\xi}(\boldsymbol{\omega}))$

$$\mathcal{L}(\mathbf{y}, \boldsymbol{\xi}(\boldsymbol{\omega}); u(\mathbf{y}, \boldsymbol{\xi}(\boldsymbol{\omega}))) = \mathcal{S}(\mathbf{y}, \boldsymbol{\xi}(\boldsymbol{\omega})), \tag{1}$$

  $\mathcal{L}$ and $\mathcal{S}$ defined on $D \times T \times \Xi$, with $D \subset \mathbb{R}^d$, $d \in \{1, 2, 3\}$, and $T \subset \mathbb{R}$. $\boldsymbol{\omega}$ denotes events in the complete probability space $(\Omega,\ \mathcal{F},\ P)$ with $\mathcal{F} \subset 2^{\Omega}$ the σ algebra of subsets of Ω and P a probability measure.

- The objective of **uncertainty propagation** is to find the probability distribution of $u(\mathbf{y}, \boldsymbol{\xi})$ and its statistical moments $\mu_{\mathrm{u}_i}(\mathbf{y})$ given by

$$\mu_{\mathrm{u}_i}(\mathbf{y}) = \int_{\Xi} u(\mathbf{y}, \boldsymbol{\xi})^i f_{\xi}(\boldsymbol{\xi}) \mathrm{d}\boldsymbol{\xi}. \tag{2}$$

Methods for uncertainty quantification

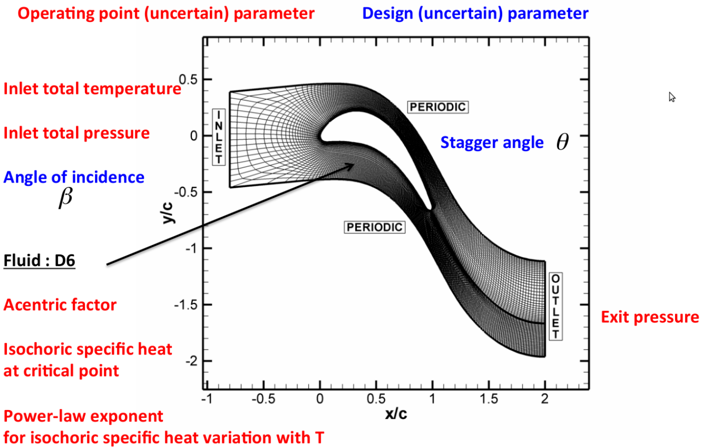
Mathematical formulation

A local UQ method computes these weighted integrals over parameter space Ξ as a summation of integrals over n_{e} disjoint subdomains $\Xi = \bigcup_{j=1}^{n_{\mathrm{e}}} \Xi_j$

$$\mu_{\mathrm{u}_i}(\mathbf{x}, \mathbf{y}, t) = \sum_{j=1}^{n_{\mathrm{e}}} \int_{\Xi_j} u(\mathbf{x}, \mathbf{y}, t, \boldsymbol{\xi})^i f_{\xi}(\boldsymbol{\xi}) \mathrm{d}\boldsymbol{\xi}. \tag{3}$$

- Various available methods : intrusive / non-intrusive
- Key issues : computational cost in high dimension, handling of mixed epistemic/aleatory uncertainty
- Present work : use of a non-intrusive **Polynomial Chaos Method**
- Epistemic uncertainty treated with a uniform pdf

Dense-gas ORC turbine



Application on a dense-gas flow in a turbine cascade

Peng-Robinson equation of state

- Thermal equation of state

$$p = \frac{RT}{v-b} - \frac{a(T,\omega)}{v^2 + 2bv - b^2}. \tag{4}$$

a and b substance-specific parameters and ω the fluid acentric factor.
Power law for the ideal-gas isochoric specific heat

$$c_{v,\infty}(T) = c_{v,\infty}(T_c)\left(\frac{T}{T_c}\right)^n \tag{5}$$

- Eqs. 4 and 5 adimensionalized depend on 3 parameters
$\rightarrow$ Three (epistemic) uncertainties on ω (2%), $c_{v\infty}(T_c)$ and n (6%) (Uniform pdf)

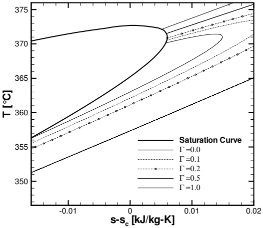
Reviewing the sources of uncertainty

Three main sources of uncertainties (globally **eight** uncertainties)

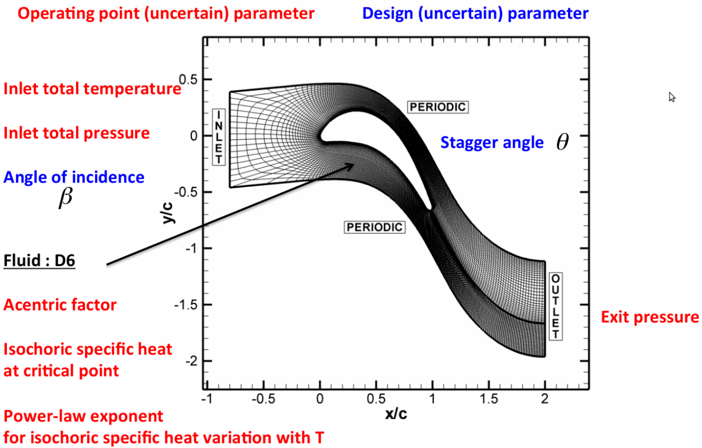
- On the **inlet** turbine conditions (aleatory), *i.e.* inlet total temperature, T_{in}/T_c, and inlet total pressure, p_{in}/p_c (3.0%)
- On the **thermodynamic** model (epistemic), *i.e.* ω (2%), $c_{v\infty}$ and n (6%)
- On **geometrical** parameters (aleatory), *i.e.* angle of incidence β (3%), stagger angle θ (3%) and the blade thickness ϕ (2%)

Interaction between thermodynamics and specific effects of dense-gases close to the saturation curve

UQ analysis is fundamental



Dense-gas ORC turbine



First type of results derived from stochastic analysis

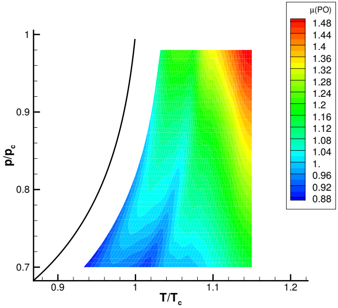
Parametric Study on inlet conditions T_{in}/T_c and p_{in}/p_c

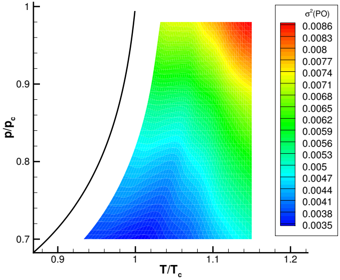
- For each couple T_{in}/T_c, p_{in}/p_c, compute mean $\mu(PO)$ and variance $\sigma(PO)$ of Power Output retained as turbine performance index
  $\rightarrow$ 1st contribution of stochastic analysis, see where performances are affected by large uncertainties
  $\rightarrow$ 2nd contribution, ANOVA analysis through TSI indexes, computation of **predominant** uncertainties
- **Remark** lower limit for temperature given by the saturation curve limit
- Uncertainty region does not cross the maximal saturation curve

p_{in}/p_c	T_{in}/T_c
0.7-0.98	SCL-1.15

Table: Intervals of parametric study

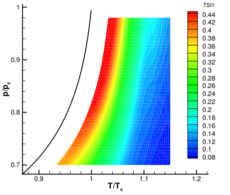
Parametric Study on inlet conditions T_{in}/T_c and p_{in}/p_c



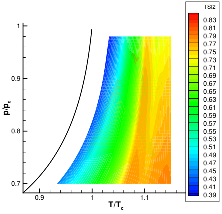


$\mu(PO)$ and $\sigma(PO)$ for each uncertainty in the plan p-T

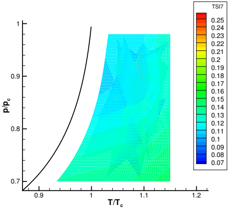
- **Performances** have to be studied in terms of $\mu(PO)$ and $\sigma(PO)$
- Where $\mu(PO)$ is higher also associated variance is high
  $\rightarrow$ which condition should be chosen ?
- Industrial needs can rely on a **prediction** of **confidence interval**
  $\rightarrow$ **robust design**, First contribution of stochastic analysis



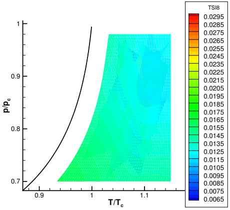
(c) p_{in}



(d) T_{in}



(e) θ

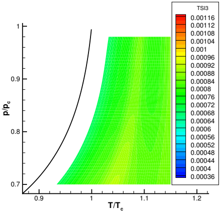


(f) ϕ

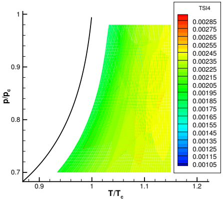
TSI contours for each uncertainty in the plan p-T
TSI $\rightarrow$ **contribution** (%) of each uncertainty to variance

- TSI associated to the uncertainty on p_{in} vary from 8% to **44%** while from 39% to **83%** for uncertainty on T_{in}
- For two geometrical parameters, θ and ϕ, TSI vary from 7% to 25% and from 0.7% to 2.9%

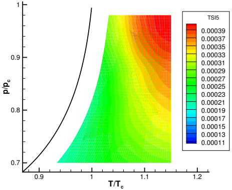
Contribution of each uncertainty to variance in the plan p-T



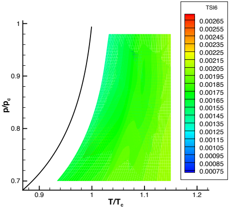
(g) ω



(h) $c_{v\infty}$



(i) n



(j) β

TSI contours for each uncertainty in the plan p-T

- TSI associated to the uncertainties on thermodynamic model and on the geometrical parameter less than 0.29% $\rightarrow$ **negligible**
  Among 8 uncertainties only 3-4 are really important
- A **hierarchy** of most influent parameters can be build
  Second contribution of stochastic analysis

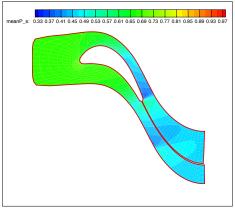
Second type of results derived from stochastic analysis

Stochastic solution for some inlet conditions

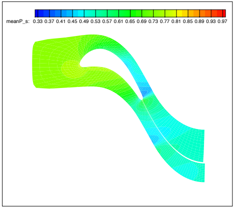
- Computation of μ and σ for the pressure coefficient
- Analysis of maximal variance region
- Physical analysis allowed by stochastic computations

Four selected designs from the parametric stochastic study

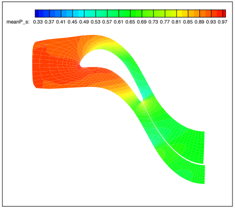
- Four designs are chosen, at lowest variance (LV), at largest mean (denoted HM), and two BT1 and BT2, for potential trade-off between $\mu(PO)$ and $\sigma(PO)$
- **Mean solutions** are sensitive to inlet conditions
- Higher inlet pressure $\rightarrow$ higher mean design
- Lower inlet temperature $\rightarrow$ lower mean design



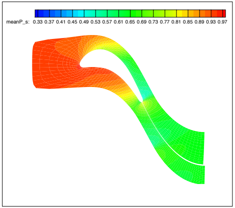
(k) LV



(l) BT2



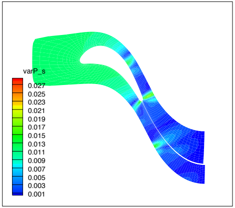
(m) BT1



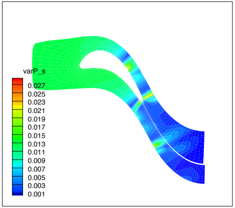
(n) HM

Four selected designs from the parametric stochastic study

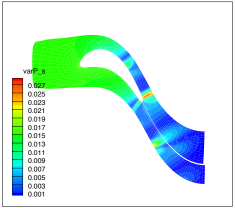
- Four designs are chosen, at lowest variance (LV), at largest mean (denoted HM), and two BT1 and BT2, for potential trade-off between $\mu(PO)$ and $\sigma(PO)$
- **Variance** concentrated on the compression shock location near the trailing edge
- Higher inlet pressure $\rightarrow$ higher variance design
- Lower inlet temperature $\rightarrow$ lower variance design



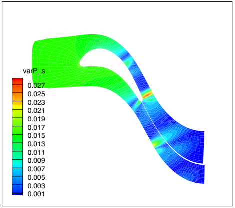
(o) LV



(p) BT2



(q) BT1



(r) HM

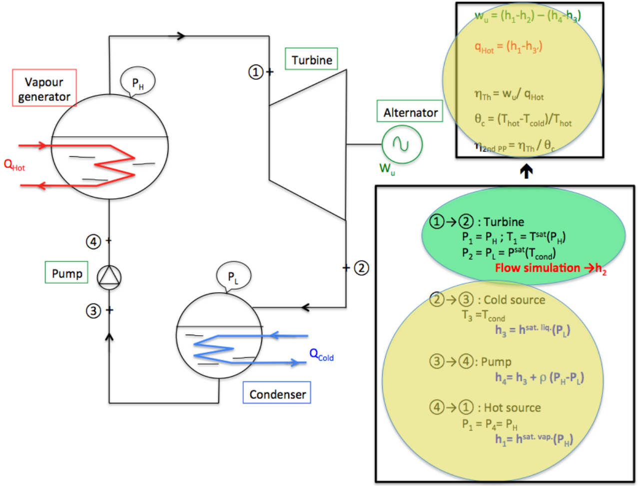
Conclusions

Numerical framework for an exhaustive analysis of ORC cycles performances

- **Efficient** stochastic method for taking into account **multiple-sources** of uncertainty
- Improved **prediction** $\rightarrow$ numerical solution with a **confidence interval**
- Application on the robust analysis of a syloxane (D_6) in a turbine cascade
- Interest of the stochastic analysis
  $\rightarrow$ **Parametric study** on the inlet conditions (For higher inlet pressure and temperature, higher mean and variance)
  $\rightarrow$ **ANOVA** analysis and contribution (%) of each uncertainty to variance (**Predominance of some uncertainties**, uncertainty on p_{in} and T_{in} are **predominant**)
  $\rightarrow$ **Stochastic analysis** of flows in turbine cascade

Numerical framework for an exhaustive analysis of ORC cycles performances

- **Global efficiency indexes** including uncertainty propagation



Thanks for your attention